Lessons from Recent Measurements of $D^0 - \overline{D}^0$ Mixing

Oram Gedalia,[1, *] Yuval Grossman,[2, †] Yosef Nir[‡,1, §] and Gilad Perez[1, ¶]

[1]*Department of Particle Physics, Weizmann Institute of Science, Rehovot 76100, Israel*

[2]*Institute for High Energy Phenomenology,*

Newman Laboratory of Elementary Particle Physics,

Cornell University, Ithaca, NY 14853, USA

Abstract

An impressive progress in measurements of the $D^0 - \overline{D}^0$ mixing parameters has been made in recent years. We explore the implications of these measurements to models of new physics, especially in view of recent upper bounds on the amount of CP violation. We update the constraints on non-renormalizable four-quark operators. We show that the experiments are close to probing minimally flavor violating models with large $\tan \beta$. The data challenge models with a scale of order TeV where the flavor violation in the down sector is suppressed by alignment and, in particular, certain classes of supersymmetric models and of warped extra dimension models.

[‡] The Amos de-Shalit chair of theoretical physics
[*]Electronic address: oram.gedalia@weizmann.ac.il
[†]Electronic address: yg73@cornell.edu
[§]Electronic address: yosef.nir,@weizmann.ac.il
[¶]Electronic address: gilad.perez@weizmann.ac.il

I. INTRODUCTION

The neutral D-meson system is the only one among the four neutral meson systems (K, D, B, B_s) that is made of up-type quarks. But this is not the only unique aspect of this system:

1. It is the only system where long distance contributions to the mixing are orders of magnitude above the (Standard Model) short distance ones.

2. It is the only system where the Standard Model (SM) contribution to the CP violation in the mixing amplitude is expected to be below the permil level.

The first point means that it is extremely difficult to theoretically predict the width- and (more significant for our purposes) mass-splitting. The second point implies that, in spite of this inherent uncertainty, $D^0 - \overline{D}^0$ mixing can unambiguously signal new physics in case that it is found to exhibit CP violation. Alternatively, if no CP violation is observed, it will provide useful constraints on new physics .

It is the purpose of this paper to obtain constraints on new physics from the new experimental data. In particular, the fact that there is now, for the first time, evidence that CP violation in $D^0 - \overline{D}^0$ mixing is small, can be cleanly interpreted in various frameworks of new physics.

The plan of this paper is as follows. In Section II we define the experimental parameters and present their allowed ranges. In Section III we define the theoretical parameters and derive their allowed ranges. In Section IV we apply the bounds to a generic effective theory, namely to $\Delta C = 2$ four quark operators, while in Section V we focus on minimally flavor violating models. Sections VI and VII deal with, respectively, supersymmetric models of alignment and models of warped extra dimensions. Our conclusions are summarized in Section VIII.

II. THE EXPERIMENTAL PARAMETERS

We start be reviewing the formalism of charm mixing (see, for example, [1, 2, 3, 4]). The two neutral D-meson mass eigenstates, $|D_1\rangle$ of mass m_1 and width Γ_1 and $|D_2\rangle$ of mass m_2 and width Γ_2, are linear combinations of the interaction eigenstates $|D^0\rangle$ (with quark

content $c\bar{u}$) and $|\overline{D^0}\rangle$ (with quark content $\bar{c}u$):

$$|D_1\rangle = p|D^0\rangle + q|\overline{D^0}\rangle, \qquad |D_2\rangle = p|D^0\rangle - q|\overline{D^0}\rangle, \tag{2.1}$$

where under CP transformation D^0 and $\overline{D^0}$ are interchanged. The average and the difference in mass and width are given by

$$m \equiv \frac{m_1 + m_2}{2}, \qquad \Gamma \equiv \frac{\Gamma_1 + \Gamma_2}{2},$$
$$x \equiv \frac{m_2 - m_1}{\Gamma}, \qquad y \equiv \frac{\Gamma_2 - \Gamma_1}{2\Gamma}. \tag{2.2}$$

The decay amplitudes into a final state f are defined as follows:

$$A_f = \langle f|\mathcal{H}|D^0\rangle, \qquad \overline{A}_f = \langle f|\mathcal{H}|\overline{D^0}\rangle. \tag{2.3}$$

We define a complex dimensionless parameter λ_f:

$$\lambda_f = \frac{q}{p}\frac{\overline{A}_f}{A_f}. \tag{2.4}$$

The time-dependent decay rates of interest are those of the doubly-Cabibbo-suppressed decay into a flavor-specific final state.

$$\Gamma[D^0(t) \rightarrow K^+\pi^-] = e^{-\Gamma t}|\overline{A}_{K^+\pi^-}|^2|q/p|^2$$
$$\times \left\{|\lambda_{K^+\pi^-}^{-1}|^2 + [\mathcal{R}e(\lambda_{K^+\pi^-}^{-1})y + \mathcal{I}m(\lambda_{K^+\pi^-}^{-1})x]\Gamma t + \frac{1}{4}(y^2 + x^2)(\Gamma t)^2\right\},$$
$$\Gamma[\overline{D^0}(t) \rightarrow K^-\pi^+] = e^{-\Gamma t}|A_{K^-\pi^+}|^2|p/q|^2$$
$$\times \left\{|\lambda_{K^-\pi^+}|^2 + [\mathcal{R}e(\lambda_{K^-\pi^+})y + \mathcal{I}m(\lambda_{K^-\pi^+})x]\Gamma t + \frac{1}{4}(y^2 + x^2)(\Gamma t)^2\right\}, \tag{2.5}$$

and singly-Cabibbo-suppressed decay into a CP eigenstate:

$$\Gamma[D^0(t) \rightarrow K^+K^-] = e^{-\Gamma t}|A_{K^+K^-}|^2\left\{1 + [\mathcal{R}e(\lambda_{K^+K^-})y - \mathcal{I}m(\lambda_{K^+K^-})x]\Gamma t\right\},$$
$$\Gamma[\overline{D^0}(t) \rightarrow K^+K-] = e^{-\Gamma t}|\overline{A}_{K^+K^-}|^2\left\{1 + [\mathcal{R}e(\lambda_{K^+K^-}^{-1})y - \mathcal{I}m(\lambda_{K^+K^-}^{-1})x]\Gamma t\right\}. \tag{2.6}$$

The expressions above are valid only in the limit $x, y \ll 1$, which is the case for the D system.

The effects of indirect CP violation can be parameterized in the following way [5]:

$$\lambda_{K^+\pi^-}^{-1} = r_d\left|\frac{p}{q}\right|e^{-i(\delta_{K\pi}+\phi)}, \qquad \lambda_{K^-\pi^+} = r_d\left|\frac{q}{p}\right|e^{-i(\delta_{K\pi}-\phi)}, \qquad \lambda_{K^+K^-} = -\left|\frac{q}{p}\right|e^{i\phi}, \tag{2.7}$$

where r_d is a real and positive dimensionless parameter, δ_f is a strong (CP conserving) mode-dependent phase, and ϕ is a weak (CP violating) universal phase. Similar expressions can be written to decays into any final state. The appearance of a single weak phase that is common to all final states is related to the absence of direct CP violation, while the absence of a strong phase in $\lambda_{K^+K^-}$ is related to the fact that the final state is a CP eigenstate.

In our analysis we assume that effects of direct CP violation are negligibly small even in the presence of new physics (NP). The question of NP contributions to direct CP violation in the doubly Cabibbo suppressed decays was investigated in detail in [6, 7] and shown to be indeed generically small. In some special cases it could reach order 30%. The singly Cabibbo suppressed decays case was studied in [8]. Typically direct CP violation is suppressed, but in special models (or corners of parameter space) it could be non-negligible. Experimental constraints on direct CP violation in charm decays were analyzed by the heavy flavor averaging group (HFAG) [9] and found to be of order one percent. Furthermore, the effect of including direct CP violation on the NP contributions was recently considered in [10] and shown to be subdominant.

The experimental measurements of the various relevant D-decay rates can be used to determine the values of the four parameters that are related to $D^0 - \overline{D^0}$ mixing: $x, y, |q/p|$ and ϕ. Impressive progress in relevant measurements has been recently achieved in the BaBar and Belle experiments. The information comes from a variety of final states of neutral D-meson decays: K^+K^-, $\pi^+\pi^-$, $K\pi^+\pi^-$, $K\ell\nu$, $K^-\pi^+$ and $K^+\pi^-$. HFAG has fitted the data, and obtained the following one sigma ranges [9]:

$$
\begin{aligned}
x &= (1.00 \pm 0.25) \times 10^{-2}, \\
y &= (0.77 \pm 0.18) \times 10^{-2}, \\
1 - |q/p| &= +0.06 \pm 0.14, \\
\phi &= -0.05 \pm 0.09,
\end{aligned}
\tag{2.8}
$$

where ϕ is given in radians. These results imply the following:

1. The width-splitting and mass-splitting are at a level close to one percent.

2. CP violation is small.

We would now like to translate these statements, made for the parameters that are used to describe the experimental results, to parameters that represent the theory input.

III. THE THEORETICAL PARAMETERS

The $\overline{D^0} - D^0$ transition amplitudes are defined as follows:

$$\langle D^0|\mathcal{H}|\overline{D^0}\rangle = M_{12} - \frac{i}{2}\Gamma_{12}, \qquad \langle \overline{D^0}|\mathcal{H}|D^0\rangle = M_{12}^* - \frac{i}{2}\Gamma_{12}^*. \tag{3.1}$$

The overall phase of the mixing amplitude is not a physical quantity. It can be changed by the choice of phase convention for the up and charm quarks. The relative phase between M_{12} and Γ_{12} is, however, phase convention independent and has therefore physics consequences. The three physical quantities related to the mixing can be defined as

$$y_{12} \equiv |\Gamma_{12}|/\Gamma, \qquad x_{12} \equiv 2|M_{12}|/\Gamma, \qquad \phi_{12} \equiv \arg(M_{12}/\Gamma_{12}). \tag{3.2}$$

Note that various papers use different sign conventions for ϕ_{12}. In the absence of direct CP violation, the following two conditions are met:

$$\mathcal{I}m(\Gamma_{12}^*\overline{A}_f/A_f) = 0, \qquad |\overline{A}_f/A_f| = 1. \tag{3.3}$$

In the following we assume that the tree level decay amplitudes in the processes (2.5) and (2.6) are given by the SM. This implies that there is no direct CP violation and that y_{12}, which is generated by decay into final states that are common to D^0 and $\overline{D^0}$ decays, is described by SM physics (see, however [11]). In that case the relations between the experimental parameters and the theoretical ones are given in Ref. [12]. Given a new physics model, one can calculate x_{12} and ϕ_{12} in terms of the model parameters. We are thus particularly interested in using experimental data to constrain x_{12} and ϕ_{12} and subsequently the new physics model parameters. Actually, the parameters that are most convenient for the analysis of NP effects are x_{12} and $x_{12}\sin\phi_{12}$ which are related directly to, respectively, the absolute value and the imaginary part of the NP operators.

We can express the theoretical parameters in terms of x, y and ϕ,

$$x_{12}^2 = \frac{x^4\cos^2\phi + y^4\sin^2\phi}{x^2\cos^2\phi - y^2\sin^2\phi},$$
$$\sin^2\phi_{12} = \frac{(x^2+y^2)^2\cos^2\phi\sin^2\phi}{x^4\cos^2\phi + y^4\sin^2\phi}, \tag{3.4}$$

or x, y and $|q/p|$,

$$x_{12}^2 = x^2\frac{(1+|q/p|^2)^2}{4|q/p|^2} + y^2\frac{(1-|q/p|^2)^2}{4|q/p|^2},$$
$$\sin^2\phi_{12} = \frac{(x^2+y^2)^2(1-|q/p|^4)^2}{16x^2y^2|q/p|^4 + (x^2+y^2)^2(1-|q/p|^4)^2}. \tag{3.5}$$

The fact that the four experimental observables $(x, y, \phi, |q/p|)$ can be expressed in terms of three theoretical input parameters $(x_{12}, y_{12}, \phi_{12})$ means that a model-independet relation between the experimental observables is predicted [12].

Using the relations between the experimental and theoretical parameters, the experimental data bound the theoretical parameters. Since the relations are complicated, we need the full correlations between the experimental measurements to perform such a task. We do not aim to do it here; Instead, we assume no correlation and use the 1σ bounds. Once a full treatment is done, our results can be straightforwardly re-scaled to the new bounds.

There are two common parameterizations that are used to present constraints on new physics from neutral meson mixing. These are the (r_D, θ_D) and the (h_D, σ_D) parameterizations defined by

$$r_D^2 = (x_{12}/x_{12}^{\rm SM}), \qquad 2\theta_D = \phi_{12} - \phi_{12}^{\rm SM}; \qquad h_D = (x_{12}^{\rm NP}/x_{12}^{\rm SM}), \qquad 2\sigma_D = \phi_{12}^{\rm NP} - \phi_{12}^{\rm SM}. \quad (3.6)$$

We use the superscript "SM" ("NP") to denote the Standard Model (New Physics) contributions. These parameterizations are, however, inappropriate for the discussion of $D^0 - \overline{D^0}$ mixing, because $x_{12}^{\rm SM}$ is poorly known. It is more useful to use, instead, the ratio $x_{12}^{\rm NP}/x_{12}$ and the phase $\phi_{12}^{\rm NP}$. In terms of the standard parameterizations, $x_{12}^{\rm NP}/x_{12} = h_D/r_D^2$ and $\phi_{12}^{\rm NP} = 2\sigma_D$ (where, for the latter equation, we use $\phi_{12}^{\rm SM} = 0$).

The most plausible mechanism that has been identified as a possible source of $y_{12} \sim 0.01$ – SU(3)-breaking in phase-space factors [13] – predicts that $x_{12}^{\rm SM} \lesssim y_{12}$ [14]. It is therefore very likely that $x_{12}^{\rm SM} \lesssim x_{12}$. Assuming that there are no accidental strong cancellations between the standard model and the new physics contributions to M_{12}, we can use the data to bound the NP parameters. The experimental data, Eq. (2.8), then give

$$x_{12}^{\rm NP} \lesssim x_{12}^{\rm exp} \sim 0.012, \qquad x_{12}^{\rm NP} \sin\phi_{12}^{\rm NP} \lesssim x_{12}^{\rm exp} \sin\phi_{12}^{\rm exp} \sim 0.0022, \qquad (3.7)$$

where $x_{12}^{\rm exp}$ and $\sin\phi_{12}^{\rm exp}$ denote the upper bounds on these theoretical parameters extracted from the experimental data (at 1σ, as explained above). We plot the constraints (3.7) in the $x_{12}^{\rm NP}/x_{12} - \phi_{12}^{\rm NP}$ plane in Fig. 1 (the allowed region is shown in grey).

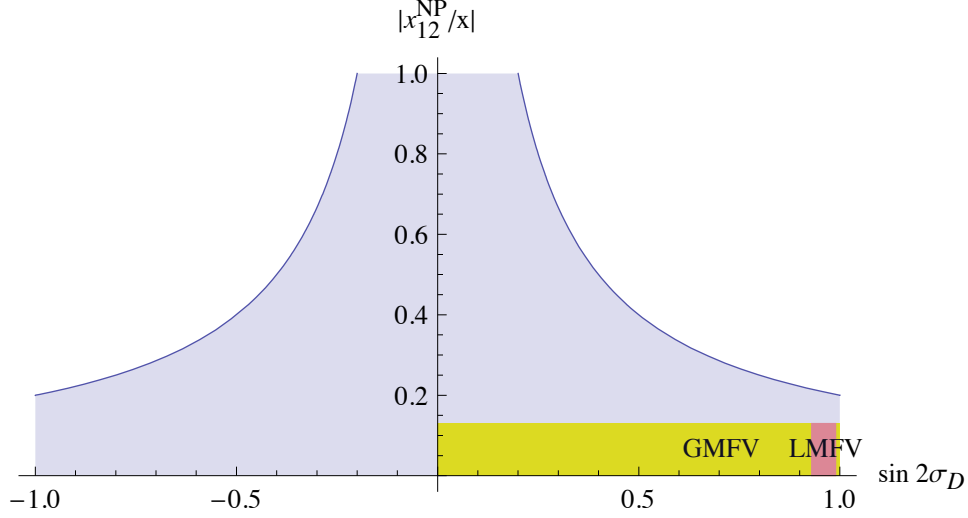We now implement these constraints in various theoretical extensions of the standard model.

FIG. 1: The allowed region, shown in grey, in the $x_{12}^{\text{NP}}/x_{12} - \sin\phi_{12}^{\text{NP}}$ plane. The pink and yellow regions correspond to the ranges predicted by, respectively, the linear MFV and general MFV classes of models (see section V for details).

IV. MODEL INDEPENDENT CONSTRAINTS

The most general effective Hamiltonian for $\Delta C = 2$ processes (see [15] for a recent discussion on the $\Delta C = 1$ case) from new physics at a high scale $\Lambda_{\text{NP}} \gg m_W$ can be written as follows:

$$\mathcal{H}_{\text{eff}}^{\Delta C=2} = \frac{1}{\Lambda_{\text{NP}}^2} \left(\sum_{i=1}^{5} z_i Q_i^{cu} + \sum_{i=1}^{3} \tilde{z}_i \tilde{Q}_i^{cu} \right), \tag{4.1}$$

where

$$\begin{aligned}
Q_1^{cu} &= \bar{u}_L^\alpha \gamma_\mu c_L^\alpha \bar{u}_L^\beta \gamma^\mu c_L^\beta, \\
Q_2^{cu} &= \bar{u}_R^\alpha c_L^\alpha \bar{u}_R^\beta c_L^\beta, \\
Q_3^{cu} &= \bar{u}_R^\alpha c_L^\beta \bar{u}_R^\beta c_L^\alpha, \\
Q_4^{cu} &= \bar{u}_R^\alpha c_L^\alpha \bar{u}_L^\beta c_R^\beta, \\
Q_5^{cu} &= \bar{u}_R^\alpha c_L^\beta \bar{u}_L^\beta c_R^\alpha,
\end{aligned} \tag{4.2}$$

and α, β are color indices. The operators $\tilde{Q}_{1,2,3}^{cu}$ are obtained from the $Q_{1,2,3}^{cu}$ by the exchange $L \leftrightarrow R$. In the following we only discuss the operators Q_i as the results for $Q_{1,2,3}$ apply to $\tilde{Q}_{1,2,3}$ as well.

We take into account the running and mixing of the operators between the scale of new

7

physics and the m_D scale. This is performed using the formula

$$\langle \overline{D^0} | \mathcal{H}_{\text{eff}}^{\Delta F=2} | D^0 \rangle_i = \sum_{j=1}^{5} \sum_{r=1}^{5} \left(b_j^{(r,i)} + \eta c_j^{(r,i)} \right) \eta^{a_j} C_i(\Lambda) \langle \overline{D^0} | Q_r^{cu} | D^0 \rangle \tag{4.3}$$

and the relevant inputs given in [16, 17].

In this way, we obtain the following constraints from $x_{12}^{\text{NP}} \lesssim 0.012$:

$$|z_1| \lesssim 5.7 \times 10^{-7} \left(\frac{\Lambda_{\text{NP}}}{1 \ TeV} \right)^2,$$

$$|z_2| \lesssim 1.6 \times 10^{-7} \left(\frac{\Lambda_{\text{NP}}}{1 \ TeV} \right)^2,$$

$$|z_3| \lesssim 5.8 \times 10^{-7} \left(\frac{\Lambda_{\text{NP}}}{1 \ TeV} \right)^2,$$

$$|z_4| \lesssim 5.6 \times 10^{-8} \left(\frac{\Lambda_{\text{NP}}}{1 \ TeV} \right)^2,$$

$$|z_5| \lesssim 1.6 \times 10^{-7} \left(\frac{\Lambda_{\text{NP}}}{1 \ TeV} \right)^2. \tag{4.4}$$

We further obtain the following constraints from $x_{12}^{\text{NP}} \sin \phi_{12}^{\text{NP}} \lesssim 0.0022$:

$$\mathcal{I}m(z_1) \lesssim 1.1 \times 10^{-7} \left(\frac{\Lambda_{\text{NP}}}{1 \ TeV} \right)^2,$$

$$\mathcal{I}m(z_2) \lesssim 2.9 \times 10^{-8} \left(\frac{\Lambda_{\text{NP}}}{1 \ TeV} \right)^2,$$

$$\mathcal{I}m(z_3) \lesssim 1.1 \times 10^{-7} \left(\frac{\Lambda_{\text{NP}}}{1 \ TeV} \right)^2,$$

$$\mathcal{I}m(z_4) \lesssim 1.1 \times 10^{-8} \left(\frac{\Lambda_{\text{NP}}}{1 \ TeV} \right)^2,$$

$$\mathcal{I}m(z_5) \lesssim 3.0 \times 10^{-8} \left(\frac{\Lambda_{\text{NP}}}{1 \ TeV} \right)^2. \tag{4.5}$$

We learn the following points (the strongest constraints correspond to maximal CP violating phases):

1. Generic new physics that contributes to the operators (4.1) at tree level with couplings of $\mathcal{O}(1)$ must lie at a very high scale, $\Lambda_{\text{NP}} \gtrsim (4 - 10) \times 10^3 \ TeV$.

2. Generic new physics that contributes to the operators (4.1) at the loop level with effective coupling of $\mathcal{O}(\alpha_2^2)$ (similar to the SM) must lie at a high scale, $\Lambda_{\text{NP}} \gtrsim (1 - 3) \times 10^2 \ TeV$.

3. New physics at or below the TeV scale must have a highly suppressed coupling, *e.g.*, $z_1 \lesssim (1-6) \times 10^{-7}$ and $z_4 \lesssim (1-6) \times 10^{-8}$. If the new physics contribution is loop-suppressed by $\mathcal{O}(\alpha_2^2)$, then the flavor suppression has to be as strong as $10^{-4} - 10^{-5}$.

V.   MINIMAL FLAVOR VIOLATION

In models of minimal flavor violation (MFV) [18, 19], the Yukawa matrices are taken to be spurions,

$$Y_u(3, \bar{3}, 1), \quad Y_d(3, 1, \bar{3}), \tag{5.1}$$

under the flavor symmetry

$$G_{\text{flavor}}^q = SU(3)_Q \times SU(3)_U \times SU(3)_D. \tag{5.2}$$

We are interested in contributions of MFV new physics to the operators (4.1). These operators generate transitions between the first two generations of up mass eigenstates. The relevant basis is then the up mass basis, where Y_u is diagonal. Since, however, $SU(3)_D$ is unbroken by the operators (4.1), all couplings z_i must involve powers of $Y_d Y_d^\dagger$. Note, however, that all operators except Q_1^{cu} break an $SU(2)_U$ subgroup of $SU(3)_U$, and therefore the corresponding z_i and $\tilde{z}_i$ are further suppressed by, at least, m_c^2/m_t^2 [19]. We conclude that, if the new physics operators arise with a similar suppression scale in MFV models, the leading contribution is to the Q_1^{cu} operator:

$$z_1 \propto [y_s^2(V_{cs}^* V_{us}) + (1 + r_{\text{GMFV}}) \times y_b^2(V_{cb}^* V_{ub})]^2 \sim \begin{cases} 1 \times 10^{-15} \zeta_1 & \text{1HDM} \\ 3 \times 10^{-8} \zeta_1 & \text{2HDM, } \tan\beta \sim m_t/m_b \end{cases} \tag{5.3}$$

where r_{GMFV} is relevant to general MFV (GMFV) models, in which the contributions from higher powers of the bottom Yukawa coupling are important and need to be resummed. In such a case, the simple relation between the contribution from the strange and bottom quarks does not apply [19]. In cases where either $\tan\beta$ is low or only the leading term in the MFV expansion is important, denoted as linear MFV (LMFV), $r_{\text{GMFV}} = 0$; Otherwise it is expected to be an order one number. We thus have

$$\begin{aligned} \zeta_1 &= e^{2i\gamma} + 2r_{sb}e^{i\gamma} + r_{sb}^2 \sim 1.7i + r_{\text{GMFV}}\left[2.4i - 1 - 0.7\,r_{\text{GMFV}}\,(1+i)\right], \\ r_{sb} &= \frac{y_s^2}{y_b^2}\left|\frac{V_{us}V_{cs}}{V_{ub}V_{cb}}\right| \sim 0.5, \end{aligned} \tag{5.4}$$

where γ is the relevant phase of the unitarity triangle. We put here $y_d^2/y_s^2 = 0$ and use $|V_{cb}V_{ub}| = 1.8 \times 10^{-4}$ for the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, $\gamma \approx 67^o$ and, within the SM, $y_b(1\ TeV) = 0.014$ [20]. 1HDM stands for single Higgs doublet models, such as the SM, while 2HDM stands for two Higgs doublet models, such as the minimal supersymmetric standard model. For the latter we assume that, for $y_b \approx y_t$, there is no suppression from the bottom Yukawa coupling, namely $y_b^4/\Lambda_{NP}^2 \approx 1/(1\ TeV)^2$. As concerns the phase of the MFV contribution, there is a clear prediction for the LMFV case, independent of whether we have a 1HDM or 2HDM and of $\tan\beta$:

$$\left.\frac{\mathcal{R}e(z_1)}{\mathcal{I}m(z_1)}\right|_{\text{LMFV}} = \left.\frac{\mathcal{R}e(\zeta_1)}{\mathcal{I}m(\zeta_1)}\right|_{\text{LMFV}} = \frac{\cos 2\gamma + 2r_{sb}\cos\gamma + r_{sb}^2}{\sin 2\gamma + 2r_{sb}\sin\gamma + r_{sb}^2} \approx 0, \qquad (5.5)$$

where we use $y_b/y_s \sim 53$ [20], $|(V_{ub}V_{cb})/(V_{us}V_{cs})| \sim 8.2 \times 10^{-4}$, and $\sin 2\gamma \sim 0.7$.

We learn the following points:

1. MFV models with two Higgs doublets can contribute to $D^0 - \overline{D^0}$ mixing up to $\mathcal{O}(0.1)$ for very large $\tan\beta$ (see also [19]).

2. Single Higgs doublet models, and two Higgs doublet models with small $\tan\beta$, contribute at $\mathcal{O}(10^{-7})$.

3. The CP violating part of these contributions is not suppressed compared to the CP conserving part.

Note that the CP violating part of these contributions is the only part that can provide a convincing signal. It can give an $\mathcal{O}(0.1)$ effect, provided that two conditions are fulfilled: First, $\tan\beta \sim m_t/m_b$ is required. Second, the new physics contribution is either tree level or, if it is loop level, logarithmically enhanced. The latter applies, for example, to supersymmetry with gauge mediation with a relatively high mediation scale (that is, not far below the grand unification scale). For $\tan\beta \sim 1$ (or single Higgs doublet) models, the new physics effects are unobeservably small. In Fig. 1 we show in pink (yellow) the range predicted by the LMFV (GMFV) class of models. The GMFV yellow band is obtained by scanning the range $r_{\text{GMFV}} \in (-1, +1)$ (but keeping the magnitude of z_1 fixed for simplicity).

VI. SUPERSYMMETRY

$D^0 - \overline{D^0}$ mixing constitutes a crucial test of supersymmetric models with alignment [17, 21, 22, 23]. The combination of $D^0 - \overline{D^0}$ mixing and $K^0 - \overline{K^0}$ mixing provides (within the supersymmetric framework) a model independent constraint on the first two generations of squark doublets [24, 25, 26]. In particular, it implies that some level of degeneracy between their masses is required.

We ignore contributions involving squarks of the third generations and $\tilde{u}_L - \tilde{u}_R$ mixing. We parameterize the flavor suppression by the factors

$$
\begin{aligned}
\delta_{LL} &= \frac{\Delta\tilde{m}^2_{Q_2 Q_1}}{\tilde{m}^2_Q}(K^{u_L}_{21} K^{u_L *}_{11}), \\
\delta_{RR} &= \frac{\Delta\tilde{m}^2_{U_2 U_1}}{\tilde{m}^2_U}(K^{u_R}_{21} K^{u_R *}_{11}), \\
\langle\delta\rangle^2 &= \delta_{LL}\delta_{RR}.
\end{aligned}
\tag{6.1}
$$

where $\tilde{m}_Q$ ($\tilde{m}_U$) is the average mass of the first two squark up-doublet (up-singlet) generations, $\Delta\tilde{m}^2_{Q_2 Q_1}$ ($\Delta\tilde{m}^2_{U_2 U_1}$) is the mass-squared difference between them and K^{u_L} (K^{u_R}) is the mixing matrix in the gluino couplings to up-quark doublet (singlet) and squark doublet (up-singlet) pairs. The contributions from the first two up-type squark generations to the various terms of Eq. (4.1) can be written as follows:

$$
\begin{aligned}
\Lambda_{\mathrm{NP}} &= \tilde{m}, \\
z_1 &= -\frac{\alpha_s^2}{216} g_1(m_{\tilde{g}}^2/\tilde{m}_Q^2)\delta_{LL}^2, \\
\tilde{z}_1 &= -\frac{\alpha_s^2}{216} g_1(m_{\tilde{g}}^2/\tilde{m}_U^2)\delta_{RR}^2, \\
z_4 &= -\frac{\alpha_s^2}{216} g_4(m_{\tilde{g}}^2/\tilde{m}^2)\langle\delta\rangle^2, \\
z_5 &= -\frac{\alpha_s^2}{216} g_5(m_{\tilde{g}}^2/\tilde{m}^2)\langle\delta\rangle^2.
\end{aligned}
\tag{6.2}
$$

Here, $\tilde{m}$ is the average squark mass, $m_{\tilde{g}}$ is the gluino mass and $g_i(x)$ are known kinematic functions [27] (for simplicity, we neglect the LR contributions),

$$
\begin{aligned}
g_1(x) &= 24x f_6(x) + 66\tilde{f}_6(x), \\
g_4(x) &= 504x f_6(x) - 72\tilde{f}_6(x), \\
g_5(x) &= 24x f_6(x) + 120\tilde{f}_6(x),
\end{aligned}
\tag{6.3}
$$

with

$$f_6(x) = \frac{6(1+3x)\log x + x^3 - 9x^2 - 9x + 17}{6(x-1)^5},$$

$$\tilde{f}_6(x) = \frac{6x(1+x)\log x - x^3 - 9x^2 + 9x + 1}{3(x-1)^5}. \tag{6.4}$$

We use as reference point $m_{\tilde{g}} = \tilde{m}$, for which

$$g_1(1) = -1, \quad g_4(1) = 138/5, \quad g_5(1) = -14/5. \tag{6.5}$$

We obtain:

$$z_1 \sim 3.7 \times 10^{-5} \delta_{LL}^2,$$

$$\tilde{z}_1 \sim 3.7 \times 10^{-5} \delta_{RR}^2,$$

$$z_4 \sim 1.0 \times 10^{-3} \langle \delta \rangle^2,$$

$$z_5 \sim 1.0 \times 10^{-4} \langle \delta \rangle^2. \tag{6.6}$$

Taking $\tilde{m} \lesssim 1\,TeV$, we find the following constrains from Eqs. (4.4) and (4.5):

$$|\delta_{LL}| \lesssim 0.13,$$

$$|\delta_{RR}| \lesssim 0.13,$$

$$|\langle \delta \rangle| \lesssim 0.008, \tag{6.7}$$

$$\left[\mathcal{I}m(\delta_{LL})^2\right]^{1/2} \lesssim 0.05,$$

$$\left[\mathcal{I}m(\delta_{RR})^2\right]^{1/2} \lesssim 0.05,$$

$$\left[\mathcal{I}m\langle \delta \rangle^2\right]^{1/2} \lesssim 0.003. \tag{6.8}$$

In models of alignment,

$$|K_{21}^{u_L} K_{11}^{u_L *}| \approx |V_{us}| \sim 0.23,$$

$$|K_{21}^{u_R} K_{11}^{u_R *}| \sim (m_u/m_c)/|V_{us}| \sim 0.009, \tag{6.9}$$

where we used $m_u/m_c \approx 0.002$. Comparing to Eq. (6.7), we find the following upper bounds on mass splittings:

$$\frac{\Delta \tilde{m}_{Q_2 Q_1}^2}{\tilde{m}_Q^2} \leq 0.56,$$

$$\left[\frac{\Delta \tilde{m}_{Q_2 Q_1}^2}{\tilde{m}_Q^2} \frac{\Delta \tilde{m}_{U_2 U_1}^2}{\tilde{m}_U^2}\right]^{1/2} \lesssim 0.17. \tag{6.10}$$

Furthermore, in models of alignment, the phases are assumed to be of order one. Taking maximal phases, we obtain from Eq. (6.8)

$$\frac{\Delta \tilde{m}^2_{Q_2 Q_1}}{\tilde{m}^2_Q} \leq 0.23,$$

$$\left[\frac{\Delta \tilde{m}^2_{Q_2 Q_1}}{\tilde{m}^2_Q} \frac{\Delta \tilde{m}^2_{U_2 U_1}}{\tilde{m}^2_U}\right]^{1/2} \lesssim 0.071. \tag{6.11}$$

Taking [28] $\tilde{m}_Q = \frac{1}{2}(\tilde{m}_{Q_1} + \tilde{m}_{Q_2})$ and similarly for the SU(2)-singlet squarks, we find that we thus have an upper bound on the splitting between the first two squark generations:

$$\frac{m_{\tilde{Q}_2} - m_{\tilde{Q}_1}}{m_{\tilde{Q}_2} + m_{\tilde{Q}_1}} \lesssim 0.05 - 0.14,$$

$$\frac{m_{\tilde{u}_2} - m_{\tilde{u}_1}}{m_{\tilde{u}_2} + m_{\tilde{u}_1}} \lesssim 0.02 - 0.04. \tag{6.12}$$

The first bound applies to the up squark doublets, while the second to the average of the doublet mass splitting and the singlet mass splitting. The range in each of the bounds corresponds to values of the phase between zero and maximal. We can thus make the following conclusions concerning models of alignment:

1. The mass splitting between the first two squark doublet generations should be below 14%. For phases of order one, the bound is about $2 - 3$ times stronger.

2. In the simplest models of alignment, the mass splitting between the first two squark generations should be smaller than about four percent.

3. The second (stronger) bound can be avoided in more complicated models of alignment, where holomorphic zeros suppress the mixing in the singlet sector.

4. While renormalization group evolution (RGE) effects can provide some level of universality, even for anarchical boundary conditions, the upper bound (6.12) requires not only a high scale of mediation [29] but also that, at the scale of mediation, the gluino mass is considerably higher than the squark masses.

In any model where the splitting between the first two squark doublet generations is larger than $\mathcal{O}(y_c^2)$, $|K^{u_L}_{21} - K^{d_L}_{21}| = \sin \theta_c = 0.23$. Given the constraints from Δm_K and ϵ_K on $|K^{d_L}_{12}|$, one arrives at a constraint very similar to the first bound in Eq. (6.12). We conclude that the constraints on the level of degeneracy between the squark doublets (stronger than

five to fourteen percent) apply to any supersymmetric model where the mass of the first two squark doublet generations is below TeV. It is suggestive that the mechanism that mediates supersymmetry breaking is flavor-universal, as in gauge mediation.

VII.   WARPED EXTRA DIMENSIONS

Randall-Sundrum (RS) models of warped extra dimensions predict that there are new tree level contributions to $D^0 - \overline{D^0}$ mixing from the exchange of Kaluza-Klein (KK) gluons [30]. The flavor suppression of such contributions is determined, up to $\mathcal{O}(1)$ uncertainties from the five-dimensional Yukawa couplings, by the values of the quark wave-functions on the IR brane, f_{Q_i} (f_{u_i,d_i}), for the left-handed (right-handed) fields. These wave-functions, which depend on the fermion bulk masses c_i, can be estimated from the quark flavor parameters,

$$|V_{ij}| \sim f_{Q_i}/f_{Q_j}, \quad y_{u_i} \sim f_{Q_i}f_{u_i}, \quad y_{d_i} \sim f_{Q_i}f_{d_i}, \tag{7.1}$$

where y_{q_i} are the four-dimensional Yukawa couplings.

A question that is crucial in many respects and, in particular, for the discovery potential of the LHC, is that of the lower bound on the mass of the KK excitations. There are bounds from electroweak precision measurements on the KK mass scale of order 3 TeV. The weakest flavor constraints arise in models where there is an alignment of the down sector flavor parameters such that contributions to flavor changing neutral currents in the down sector are highly suppressed [31, 32, 33]. In that case, the bounds from $D^0 - \overline{D^0}$ mixing play a crucial role, since the up sector tends to possess an anarchical structure as in the generic models [30].

We first consider the case where the Higgs is localized on the IR brane. The KK gluon exchange contributes to the various terms of Eq. (4.1) by mediating four-quark interactions:

$$\begin{aligned}
\Lambda_{\mathrm{NP}} &= M_G, \\
z_1 &\sim \frac{g_{s*}^2}{6}\gamma(c_{Q_2})^2(V_{ub}V_{cb}^*)^2 f_{Q_3}^4, \\
z_4 &\sim \frac{g_{s*}^2}{Y_*^2}\gamma(c_{Q_2})\gamma(c_{u_2})y_u y_c,
\end{aligned} \tag{7.2}$$

where we listed the two operators that yield the strongest constraints. Here M_G is the mass of the KK gluon, g_{s*} is the bulk SU(3) gauge coupling, Y_* is the typical size of the

TABLE I: The z_i parameters in a generic RS model. The bound on Y_* is for $M_G = 3$ TeV. Both bounds correspond to maximal phase, and are relaxed by a factor ~ 2.4 for a vanishing phase.

Parameter	Numerical estimate	$(M_G)_{\min}[\text{TeV}]$	$(Y_*)_{\min}$
z_1	$5.8 \times 10^{-7} f_{Q_3}^4$	$0.73 f_{Q_3}^2$	-
z_4	$2.2 \times 10^{-7}/Y_*^2$	$4.9/Y_*$	1.6

(presumably anarchical) entries in the IR brane localized Yukawa interactions and $\gamma(c)$ is a correction function to the overlap of the quarks with the first KK gluon, given by [33, 34]

$$\gamma(c) = \frac{\sqrt{2}}{J_1(x_1)} \frac{0.7}{6 - 4c} \left(1 + e^{c/2}\right) , \tag{7.3}$$

with $x_1 \approx 2.4$ being the first root of the Bessel function $J_0(x_1) = 0$.

For the purpose of a quantitative analysis, we take $g_{s*} = 3$, obtained by matching to the 4D coupling at one-loop [35]. We also use $c_{Q_2} = 0.58$ and $c_{u_2} = 0.53$, which are reasonable representative values for the relevant range of the parameters f_{Q_3} and Y_* (in which the result can change by only a few percent). The value of $f_{Q_3} = 0.4\,(\sqrt{2})$ represents a profile that is fairly flat, $c_{Q_3} = 0.42$ (sharply localized in IR, $c_{Q_3} = -0.5$) [38]. In any case, our strongest constraint, which comes from z_4, is independent of f_{Q_3}. For the quark sector parameters, we use

$$y_u = 6.1 \times 10^{-6}, \quad y_c = 2.95 \times 10^{-3}, \quad |V_{ub}V_{cb}^*| = 1.6 \times 10^{-4}, \tag{7.4}$$

where we evaluate $m_{u,c}$ at 3 TeV with SM-RGE, given in Ref. [20] (the experimental bounds are also calculated at 3 TeV), and use the central values of the CKM elements given in Ref. [36]. We present our results for the lower bound on the KK scale (that is, the mass of the KK gluon) in Table I. This analysis can be used to obtain a lower bound on Y_*, assuming $M_G = 3$ TeV, which is the minimal value allowed by electroweak precision constraints. This is also shown in Table I.

Note that in Ref. [26], a much stronger bound was derived based on the same experimental data. This difference is the result of a combination of two $\mathcal{O}(1)$ factors: First, g_{s*} was taken to be 6 and not 3 (based on tree-level instead of one-loop matching), and second the CKM factor was roughly evaluated by λ_C^5, instead of $|V_{ub}V_{cb}^*|$ in the current paper (the latter is more appropriate in the context of constraining models with alignment in the down sector). These

differences reflect the fact that $\mathcal{O}(1)$ uncertainties are always present in the RS framework, as a result of its limited predictive power.

The bounds in Table I can be further relaxed by considering a bulk Higgs (note that the dimension of the Yukawa coupling changes in this case, so that Y_* is given in units of $\sqrt{k}$). The couplings of the light quarks with a bulk Higgs are *enhanced*. The RS contribution to z_1 does not change, to leading order, since the effect of a bulk Higgs on the rotation angles f_i/f_j is subleading. However, the mass relations of the form $y_i \approx Y_* f_{Q_i} f_{u_i}$, used to obtain the contribution to z_4, are altered in this case. This change can be expressed by a function that corrects for the overlap of the wavefunctions of the Higgs with two zero-mode quarks, relative to the IR Higgs case [37]:

$$y_i \approx Y_* f_{Q_i} f_{u_i} r_{00}^H(\beta, c_{Q_i}, c_{u_i}), \qquad r_{00}^H(\beta, c_{Q_i}, c_{u_i}) = \frac{\sqrt{2(1+\beta)}}{2 + \beta - c_{Q_i} - c_{u_i}}, \qquad (7.5)$$

where r_{00}^H depends on the Higgs profile in the bulk, parameterized by $\beta = \sqrt{4 + \mu^2}$ (μ is the bulk mass of Higgs in units of k). Hence, z_4 is given in this case by

$$z_4 \sim \frac{g_{s*}^2}{Y_*^2} \frac{\gamma(c_{Q_2})\gamma(c_{u_2})}{r_{00}^H(\beta, c_{Q_1}, c_{u_1}) r_{00}^H(\beta, c_{Q_2}, c_{u_2})} y_u y_c \,. \qquad (7.6)$$

For a Higgs maximally spread into the bulk (that is, saturating the anti-de Sitter stability bound – $\beta = 0$), the bound on M_G is reduced by a factor of ~ 2, to $2.4/Y_*$ TeV.

We learn that the recent measurements of $D^0 - \overline{D^0}$ mixing impose additional constraints on the RS model. In particular, given an IR Higgs, a 3 TeV KK scale requires $Y_* \gtrsim 1.6$, which is close to the perturbativity bound $Y_* \lesssim 2\pi/N_{\mathrm{KK}}$, where N_{KK} stands for the number of KK state below the theory's UV cutoff. That has also implications for alignment models [31], where the larger the value of Y_* is, the larger the next to leading order corrections are, which spoils the alignment.

VIII.   CONCLUSIONS

Recent bounds on CP violation in $D^0 - \overline{D^0}$ mixing are particularly significant, because – unlike the mass splitting and the width splitting – there is no standard model contribution that can interfere with the new physics. We studied the implications of these measurements to various frameworks of new physics.

For generic models, we obtained the following results:

- Generic new physics that contributes to the operators (4.1) at tree level with couplings of $\mathcal{O}(1)$ must lie at a very high scale, $\Lambda_{\mathrm{NP}} \gtrsim (4 - 10) \times 10^3 \; TeV$.

- Generic new physics that contributes to the operators (4.1) at the loop level with effective coupling of $\mathcal{O}(\alpha_2^2)$ (similar to the SM) must lie at a high scale, $\Lambda_{\mathrm{NP}} \gtrsim (1 - 3) \times 10^2 \; TeV$.

- New physics at or below the TeV scale must have a highly suppressed coupling, $e.g.$, $z_1 \lesssim (1 - 6) \times 10^{-7}$ and $z_4 \lesssim (1 - 6) \times 10^{-8}$. If the new physics contribution is loop-suppressed by $\mathcal{O}(\alpha_2^2)$, then the flavor suppression has to be as strong as $10^{-4} - 10^{-5}$.

- Neither electroweak loop suppression nor alignment of order $\sin \theta_c$ are sufficient to allow new physics at the TeV scale. There must be some level of degeneracy – stronger than $\mathcal{O}(0.1)$ – to allow that.

For models with minimal flavor violation (MFV), we reached the following conclusions:

- MFV models with two Higgs doublets can contribute to $D^0 - \overline{D^0}$ mixing up to $\mathcal{O}(0.1)$ of the experimental value for very large $\tan \beta$.

- Single Higgs doublet models, and two Higgs doublet models with small $\tan \beta$, contribute at $\mathcal{O}(10^{-7})$.

- The CP violating part of these contributions is not suppressed compared to the CP conserving part.

Our findings imply that MFV models with very large $\tan \beta$ will be probed once the experimental sensitivity to CP violation in mixing reaches the ten percent level.

For supersymmetric models with quark-squark alignment, we learn the following:

- The mass splitting between the first two squark doublet generations should be below 14%. For phases of order one, the bound is two to three times stronger.

- In the simplest models of alignment, the mass splitting between the first two squark generations should be smaller than about four percent.

- The second (stronger) bound can be avoided in more complicated models of alignment where holomorphic zeros suppress the mixing in the singlet sector.

- While RGE effects can provide some level of universality, even for anarchical boundary conditions, the upper bound (6.12) requires not only a high scale of mediation [29] but also that, at the scale of mediation, the gluino mass is considerably higher than the squark masses [26].

For models of warped extra dimensions where alignment is used to relax the bounds from $K^0 - \overline{K^0}$ mixing, we find the following:

- The lower bound on the KK-gluon mass is pushed to be above 2.5 (10) TeV for the maximally (minimally) composite top case depending on the size of the 5D Yukawa coupling assuming at least three KK states. The flavor constraints are then stronger than those from the electroweak precision measurements for a large portion of the parameter space.

Acknowledgements

---

[1] G. Blaylock, A. Seiden and Y. Nir, Phys. Lett. B **355**, 555 (1995) [arXiv:hep-ph/9504306].

[2] E. Golowich, J. Hewett, S. Pakvasa and A. A. Petrov, Phys. Rev. D **76**, 095009 (2007) [arXiv:0705.3650 [hep-ph]].

[3] S. Bianco, F. L. Fabbri, D. Benson and I. Bigi, Riv. Nuovo Cim. **26N7**, 1 (2003) [arXiv:hep-ex/0309021].

[4] I. I. Bigi, M. Blanke, A. J. Buras and S. Recksiegel, JHEP **0907**, 097 (2009) [arXiv:0904.1545 [hep-ph]].

[5] S. Bergmann, Y. Grossman, Z. Ligeti, Y. Nir and A. A. Petrov, Phys. Lett. B **486**, 418 (2000) [arXiv:hep-ph/0005181].

[6] S. Bergmann and Y. Nir, JHEP **9909**, 031 (1999) [arXiv:hep-ph/9909391].

[7] G. D'Ambrosio and D. N. Gao, Phys. Lett. B **513**, 123 (2001) [arXiv:hep-ph/0105078].

[8] Y. Grossman, A. L. Kagan and Y. Nir, Phys. Rev. D **75**, 036008 (2007) [arXiv:hep-ph/0609178].

[9] E. Barberio *et al.* [Heavy Flavor Averaging Group], arXiv:0808.1297 [hep-ex], and online update at http://www.slac.stanford.edu/xorg/hfag.

[10] A. L. Kagan and M. D. Sokoloff, arXiv:0907.3917 [hep-ph].

[11] E. Golowich, S. Pakvasa and A. A. Petrov, Phys. Rev. Lett. **98**, 181801 (2007) [arXiv:hep-ph/0610039].

[12] Y. Grossman, Y. Nir and G. Perez, Phys. Rev. Lett. **103**, 071602 (2009) [arXiv:0904.0305 [hep-ph]].

[13] A. F. Falk, Y. Grossman, Z. Ligeti and A. A. Petrov, Phys. Rev. D **65**, 054034 (2002) [arXiv:hep-ph/0110317].

[14] A. F. Falk, Y. Grossman, Z. Ligeti, Y. Nir and A. A. Petrov, Phys. Rev. D **69**, 114021 (2004) [arXiv:hep-ph/0402204].

[15] E. Golowich, J. Hewett, S. Pakvasa and A. A. Petrov, Phys. Rev. D **79**, 114030 (2009) [arXiv:0903.2830 [hep-ph]].

[16] M. Bona *et al.* [UTfit Collaboration], JHEP **0803**, 049 (2008) [arXiv:0707.0636 [hep-ph]].

[17] M. Ciuchini, E. Franco, D. Guadagnoli, V. Lubicz, M. Pierini, V. Porretti and L. Silvestrini, Phys. Lett. B **655**, 162 (2007) [arXiv:hep-ph/0703204].

[18] See *e.g.:* G. D'Ambrosio, G. F. Giudice, G. Isidori and A. Strumia, Nucl. Phys. B **645**, 155 (2002) [arXiv:hep-ph/0207036]; L. J. Hall and L. Randall, Phys. Rev. Lett. **65**, 2939 (1990); R. S. Chivukula and H. Georgi, Phys. Lett. B **188**, 99 (1987); A. J. Buras, P. Gambino, M. Gorbahn, S. Jager and L. Silvestrini, Phys. Lett. B **500**, 161 (2001) [arXiv:hep-ph/0007085]; A. Ali and D. London, Eur. Phys. J. C **9**, 687 (1999) [arXiv:hep-ph/9903535]; E. Gabrielli and G. F. Giudice, Nucl. Phys. B **433**, 3 (1995) [Erratum-ibid. B **507**, 549 (1997)] [arXiv:hep-lat/9407029].

[19] A. L. Kagan, G. Perez, T. Volansky and J. Zupan, arXiv:0903.1794 [hep-ph].

[20] Z. z. Xing, H. Zhang and S. Zhou, Phys. Rev. D **77**, 113016 (2008) [arXiv:0712.1419 [hep-ph]].

[21] Y. Nir and N. Seiberg, Phys. Lett. B **309**, 337 (1993) [arXiv:hep-ph/9304307].

[22] M. Leurer, Y. Nir and N. Seiberg, Nucl. Phys. B **420**, 468 (1994) [arXiv:hep-ph/9310320].

[23] Y. Nir, JHEP **0705**, 102 (2007) [arXiv:hep-ph/0703235].

[24] Y. Nir, arXiv:0708.1872 [hep-ph].

[25] J. L. Feng, C. G. Lester, Y. Nir and Y. Shadmi, Phys. Rev. D **77**, 076002 (2008) [arXiv:0712.0674 [hep-ph]].

[26] K. Blum, Y. Grossman, Y. Nir and G. Perez, Phys. Rev. Lett. **102**, 211802 (2009) [arXiv:0903.2118 [hep-ph]].

[27] M. Ciuchini *et al.*, JHEP **9810**, 008 (1998) [arXiv:hep-ph/9808328].

[28] G. Raz, Phys. Rev. D **66**, 037701 (2002) [arXiv:hep-ph/0205310].

[29] Y. Nir and G. Raz, Phys. Rev. D **66**, 035007 (2002) [arXiv:hep-ph/0206064].

[30] K. Agashe, G. Perez and A. Soni, Phys. Rev. D **71**, 016002 (2005) [arXiv:hep-ph/0408134].

[31] A. L. Fitzpatrick, G. Perez and L. Randall, Phys. Rev. Lett. **100**, 171604 (2008) [arXiv:0710.1869 [hep-ph]]; J. Santiago, JHEP **0812**, 046 (2008) [arXiv:0806.1230 [hep-ph]]; C. Csaki, G. Perez, Z. Surujon and A. Weiler, arXiv:0907.0474 [hep-ph].

[32] C. Csaki, A. Falkowski and A. Weiler, Phys. Rev. D **80**, 016001 (2009) [arXiv:0806.3757 [hep-ph]].

[33] C. Csaki and D. Curtin, Phys. Rev. D **80**, 015027 (2009) [arXiv:0904.2137 [hep-ph]].

[34] C. Csaki, A. Falkowski and A. Weiler, JHEP **0809**, 008 (2008) [arXiv:0804.1954 [hep-ph]].

[35] K. Agashe, A. Azatov and L. Zhu, Phys. Rev. D **79**, 056006 (2009) [arXiv:0810.1016 [hep-ph]].

[36] C. Amsler *et al.* [Particle Data Group], Phys. Lett. B **667**, 1 (2008).

[37] O. Gedalia, G. Isidori and G. Perez, arXiv:0905.3264 [hep-ph].

[38] We thank Kaustubh Agashe for pointing out an inaccuracy in these values in the previous version of the paper.